



**AN ANALYSIS OF ROBUST WORKFORCE  
SCHEDULING MODELS FOR A NURSE  
ROSTERING PROBLEM**

THESIS

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AFIT/GLM/ENS/07-12

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## Abstract

Disruptions impacting workforce schedules can be costly. A 1999 study of the United Kingdom's National Health Service estimated that as much as four percent of the total resources spent on staffing were lost to schedule disruptions like absenteeism. Although disruptions can not be eliminated, workforce schedules can be improved to be more responsive to disruptions. One key area of study that has expanded over the past few years is the application of traditional scheduling techniques to re-rostering problems. These efforts have provided methods for responding to schedule disruptions, but typically require deviations to the disrupted schedule.

This thesis examines five workforce scheduling models designed for a nurse rostering problem. Each model is designed to produce a robust workforce schedule that remains valid in the midst of disruptions and requires no schedule deviations. Each model is evaluated based on the number of disruptions it can receive before becoming invalid. Nonparametric statistical analysis is used to analyze the disruption data for each model and determine which workforce scheduling model produces the most robust schedule. The results of this research indicate that additional manpower must be applied to the correct skill sets in order to produce robust workforce schedules. Furthermore, workforce managers can consider leaving a portion of the workforce unscheduled (or in reserve) to accommodate schedule disruptions.

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*To My Loving Wife*

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Paul K. Tower

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# AN ANALYSIS OF ROBUST WORKFORCE SCHEDULING MODELS FOR A NURSE ROSTERING PROBLEM

## I. Introduction

### Background

Absenteeism of professional caregivers in the healthcare industry is very costly. In a 1999 study of the United Kingdom's National Health Service (NHS), absenteeism had a dramatic effect on the budget allocated for workforce staffing (Ritchie et al, 1999: 702). An estimated 70% of the NHS's total resources were spent on staffing. Some absenteeism rates in the NHS healthcare system were as high as 6%. Therefore, as much as 4% of the NHS budget lost due to absenteeism. Absenteeism continues to be an important topic throughout personnel management literature. Although healthcare managers across the industry are striving to understand the cause of absenteeism and reduce its effect on healthcare workforces (Eriksen et al., 2003: 271; Whitehead, 2006: 61), these efforts will not eliminate absenteeism. Therefore, it is imperative that schedulers work to improve workforce scheduling by developing schedules that are responsive to absenteeism and other disruptions.

### Overview

A classical workforce scheduling problem consists of assigning employees to shifts in a continuous operations environment. A continuous operation is an operation

that requires a workforce to be available 24 hours a day, over some subset of days (Knighton, 2005: 86). Organizations that function under continuous operations face a unique challenge with respect to managing workforce demand requirements. The goal of a scheduler is to satisfy demand requirements per shift while feasibly or optimally solving a greater objective. This overarching objective could be minimizing the total cost of wages for all employees scheduled, maximizing the value of the schedule based on employee or organizational preferences, or distribute shifts evenly across all employees (Ernst et al, 2004: 3). Often times the workday is broken down into three separate 8-hour shifts; which are labeled, Morning, Evening, and Night (Knighton, 2005: 88)

Scheduling under continuous operations is complicated when a scheduler must consider a large workforce with varying skills sets, personnel availabilities, and wage rates. A workforce with these varying employee factors is commonly referred to as a heterogeneous workforce (Knighton, 2005: 85). When scheduling a heterogeneous workforce, the scheduler must consider and properly weigh the various factors present in the workforce. Each of these heterogeneous factors adds constraints to the scheduling model; thereby increasing the complexity of the model.

As the size of the workforce increases and complex constraints such as varying skill sets are added to the scheduling problem, the schedule becomes less responsive to disruptions. A disruption is defined as an instance when an employee that is scheduled to work a specific shift is unavailable to work the specified shift. To fill the vacant shift requirement, the scheduler must reassign an unscheduled employee to cover the shift, while continuing to meet current demand and workforce requirements and constraints.

Two classical scheduling problems that consider heterogeneous workforces supporting continuous operations are the nurse rostering problem and the nurse rerostering problem. The nurse rostering problem has received much attention in the staff scheduling literature (Moz and Pato, 2007: 668). Staff scheduling and rostering have been studied and documented by various researchers including Cheang et al. (2003), Siferd and Benton (1992), and Ernst et al. (2004). Several models exist for rostering a staff schedule given a defined set of constraints. However, when a disruption occurs, a schedule developed using any of these models is no longer valid. The scheduler must consider changing the schedule to accommodate the disruption in the workforce. If there is an excess number of employees available to work when the disruption occurs, then rebuilding the schedule is fairly easy. However, if there is no excess of available employees, then the scheduler must reassign the workforce to each shift (also known as re-rostering).

### **Summary of Current Knowledge**

The nurse re-rostering problem has received limited attention in the current literature. Knighton (2005) considered a network-based mathematical programming approach to the re-rostering problem. Using this approach, Knighton was able to re-roster the schedule with minimum deviations to the original schedule. Knighton's model requires the scheduler to develop a constraint set that defines that "all employees are scheduled only when available, have adequate skill level, meet minimum shifts per week, and have adequate rest between shifts." (Knighton, 2005: 93) Furthermore, the scheduler must input the employee rankings, as defined by the manager, and the

employee shift preferences. Most recently, Moz and Pato (2007) developed constructive heuristics and genetic algorithms to re-roster a schedule following a disruption. Their use of genetic algorithms greatly improved the re-rostering ability of constructive heuristics alone.

Although the re-rostering problem has received attention over the past few years, the idea of developing a robust roster has received far less attention. A robust schedule is defined as a schedule that anticipates disruptions and has a predefined solution for addressing disruptions. Obviously, it would be very difficult to develop a schedule that is robust against all disruptions. However, it is feasible to develop a schedule that is robust against disruptions on the days requiring the greatest employee demand, possibly at some greater cost over an undisrupted optimal schedule. Therefore, a robust schedule is better defined as a schedule that proactively responds to disruptions to shifts with the greatest employee demand.

## **Research Problem**

The purpose of this research work is to develop and identify new scheduling models that provide improved workforce schedules. The models must meet the same demand requirements of the original workforce schedule. However, in contrast to the original re-rostering problem, the objective of the new model is to evenly distribute excess employee availability in order to maximize the number of disruptions the schedule can receive and still remain a valid schedule. This body of research will identify the best method for building a robust workforce schedule.

The following investigative questions will aid in developing and evaluating the new scheduling model:

- IQ1: What methods are currently available for workforce scheduling?
- IQ2: What methods are available for building robust schedules in other scheduling applications?
- IQ3: Which robust workforce scheduling method provides the schedule that can respond to the greatest number of disruptions?

### **Study Delimitations**

This research focuses on identifying scheduling models that provide a workforce schedule that is robust against disruptions. To test the validity of the model, a case study is used involving nurses scheduled at a private nursing home in Maine (Oliver, 2006). Although the data is not representative of all continuous heterogeneous scheduling operations, it is assumed to be sufficient for validating the model. In contrast, this research does not address the scheduling rostering problem. This area of research has received fair, but limited treatment over the past few years (Knighton, 2005; Moz and Pato, 2005).

### **Approach and Methodology**

An integer based mathematical program is used to solve the continuous heterogeneous workforce scheduling problem. Several models are developed and evaluated based on each model's ability to respond to schedule disruptions. Schedule disruptions are randomly generated, and the model is evaluated on whether the schedule



is still valid and able meet minimum shift demand requirements. If a sufficient number of workers are still available to meet shift demand, then the model is considered robust against that level of disruptions. The model is also evaluated based on the factor (i.e. skill or manpower) causing the model to fail. This allows further comparison across workforce scheduling models.

### **Assumptions**

A few key assumptions are critical to the success of this study. The first assumption is that there is a value associated with maximizing the number of disruptions a workforce schedule can undertake and still remain valid. Furthermore, it is assumed that the value of maximizing the robustness of a schedule is greater than the value of using an optimal solution.

### **Expected Results and Future Applications**

Although there has been limited study in the area of robust scheduling with respect to continuous heterogeneous workforce scheduling, this research identifies the significant benefits of robust scheduling. First, robust scheduling minimizes the number of scheduling deviations required to address disruptions. Second, robust scheduling allows the scheduler to take a proactive approach in addressing unexpected disruptions, before they occur. Ideally, this approach will prevent a scheduler from needing to apply a re-rostering algorithm to a disrupted schedule. Finally, robust scheduling has the potential of reducing the number of deviations employees experience in their work schedules. By reducing (and possibly eliminating all-together) these deviations, an

employer can potentially save money associated with rescheduling employees. Finally, reducing the number of weekly schedule deviations should inherently prevent any employee dissatisfaction with a volatile work schedule that does not proactively address the potential for deviations.

## **II. Literature Review**

### **Introduction**

Current workforce scheduling literature addresses several concerns regarding workforce scheduling. Several models have been developed for optimizing workforce schedules when varying the number for required workdays per week and varying the number of allowable days off per week (Burns et al, 1998). Models have also been developed which incorporate varying workforce skill sets. One of the areas that requires more attention is that of re-rostering and robust workforce scheduling. Presently, Moz and Pato (2007) and Knighton (2005) are the few researchers examining the area of re-rostering within continuous heterogeneous operations. Robust workforce scheduling literature is absent from the area of continuous heterogeneous operations. However, the concept can be found in literature regarding aircrew scheduling in the airline industry (Shebalov and Klabjan, 2006).

This literature review will first present an overview of workforce scheduling and the nurse rostering problem. It will examine current scheduling techniques and research accomplished in the area of re-rostering. A review of robust scheduling is also presented. Particular attention is given to the advancement of robust scheduling in the airline industry. Finally, a brief overview of Shebalov and Klabjan's robust aircrew scheduling theory will be presented as a stepping stone for building robust workforce schedules in continuous heterogeneous operations. Although robust scheduling is absent from the current workforce scheduling literature, there is sufficient evidence for increased studies in this field.

## **Overview**

Workforce scheduling is commonly referred to as staff or personnel scheduling or rostering. It is the process of constructing work schedules for a staff of employees so that an organization can satisfy demand for its goods and services (Ernst et al, 2004: 3).

Employees are assigned to shifts at varying times in order to satisfy demand requirements. Typically, the number of employees assigned to each shift is governed by industrial regulations or other local laws. Therefore, the objective of a workforce scheduler is to schedule employees to meet shift demands, while minimizing the total cost of wages for all employees scheduled, maximizing the value of the schedule based on employee or organizational preferences, or distributing shifts evenly across all employees.

## **Workforce Scheduling Process**

Several decision support tools are available to the personnel scheduler. Schedulers can use various computer software packages for scheduling. These packages range from simple spreadsheets to complex mathematical models that use highly-developed algorithms to produce optimal or feasible solutions (Ernst et al, 2004: 3). In every case, the scheduler must follow three steps (Ernst et al, 2004: 4).

The first step is that the scheduler must determine the staff leveling required based on the service being provided. This can be determined using historical data and forecasting techniques or by examining industry standards and regulations. The second step is that the scheduler must determine the appropriate model and technique available

for building the schedule based on demand, wage rates, and shift preferences. The final step is that the scheduler must build and publish the schedule.

One key aspect of the first step is identifying the required staff leveling. Staff leveling can be defined by task-based demand, flexible demand, or shift-based demand (Ernst et al, 2004: 5). In task-based demand, personnel are scheduled to complete a predetermined list of tasks or actions. In flexible demand, demand for workers is dependent on future incidents. The arrival of future incidents is unknown, but forecasting techniques can be applied to determine the likelihood of when future incidents will occur and schedule the workforce accordingly. One example of flexible demand is determining the workforce schedule for a police department. Although it is not possible to determine the future incidents of crime, it is possible to determine the likelihood of crime occurring at certain times of the day. The police workforce can then be scheduled accordingly. The final demand type is shift-based demand. Shift-based demand is demand that is determined based on laws or industry regulations. For example, the U.S. nursing industry requires a set level of nurses to be on duty depending on the number of patient beds in the facility and the time of day.

### **The Nurse Rostering Problem**

One particular rostering problem that has received much attention throughout the past four decades is the nurse rostering problem. Cheang (2003) provides a bibliographic survey of this problem. In the nurse rostering problem, nurses must be scheduled to cover shift demand based on the number of patients or beds assigned to the hospital. The nurse rostering problem is a difficult problem in workforce scheduling because hospitals

are typically staffed 24 hours a day, seven days a week, every week of the year.

Therefore, nurses must be scheduled continuously and attention must be given to rest periods.

The nurse rostering problem can be described in one of three ways: a nurse-day view, a nurse-task view, or a nurse-shift pattern view (Cheang et al, 2003: 448). In the nurse-day view, the decision variable is indexed for each nurse and each day. The variable can take on a number of values based on the nurse's assignment for that day. Some of the values may include day shift (D), evening shift (E), night shift (N), day-off (O), and vacation leave (VL). Further indexing can be added to the nurse-day view to include indexes for individual shifts or individual skills sets. In the nurse-task view, the decision variable is indexed for each nurse and each task that the nurse will accomplish in the scheduling period. This decision variable may only assume a value of 1 if the nurse is assigned to the task, or 0 otherwise. In the nurse-shift pattern view, the decision variable is indexed for each nurse and each pattern of shifts available.

The nurse rostering problem is always governed by some set of constraints. These constraints can be defined as hard or soft constraints (Cheang et al, 2003: 449). A hard constraint is a constraint that must be satisfied. For example, minimum shift staffing requirements must be satisfied in accordance with industry regulations or local laws. A soft constraint is a constraint that is "usually involved with time requirements on personal schedules" (Cheang et al, 2003: 449). Cheang (2003) provides a list of commonly occurring constraints typically associated with the nurse rostering problem:

1. Nurses workload (minimum/maximum);
2. Consecutive same working shift (minimum/maximum/exact number);
3. Consecutive working shift/days (minimum/maximum/exact number);

4. Nurse skill levels and categories;
5. Nurses' preferences or requirements;
6. Nurses free days (minimum/maximum/consecutive free days);
7. Free time between working shifts (minimum);
8. Shift type(s) assignments (maximum shift type, requirements for each shift types);
9. Holidays and vacations (predictable);
10. Working weekend;
11. Constraints among groups/types of nurses, e.g., nurses not allowed to work together or nurses who must work together;
12. Shift patterns;
13. Historical record, e.g., previous assignments;
14. Other requirements in a shorter or longer time period other than the planning time period, e.g., every day in a shift must be assigned;
15. Constraints among shift, e.g., shifts cannot be assigned to a person at the same time.
16. Requirements of (different types of) nurses or staff demand for any shift (minimum/maximum/exact number).

Cheang also provides an overview of where further literature can be found for each constraint.

One final aspect of the nurse rostering problem is the objective function. The objective functions can be defined as minimizing a penalty cost associated with nurses working particular shifts. The function may call for maximizing the value associated with nurses' preference for particular shifts. Regardless of the objective function, the nurse rostering problem typically strives to achieve an optimal solution (Cheang et al, 2003: 450).

### **Nurse Rostering Techniques**

Ernst (2004) presents a comprehensive over view of the nurse rostering problem and its development. In the 1970s and 1980s, support tools were developed to reduce the burden on schedulers to develop workforce schedules manually. Problem constraints

were identified and techniques such as mathematical programming, goal programming, and iterative algorithms were developed and applied to develop optimal work schedules.

In the 1990s, the nurse rostering problems began to be classified under nurse rostering systems. Each system of problems had several methods associated that would aid a scheduler in producing optimal work schedules. Advances were made in applying linear programming, integer programming, network optimization techniques, and constraint programming. However, several of these solutions were limited to the case for which they were developed. The solution would require significant rework to be applied to another case (Ernst et al, 2004: 12).

Highly sophisticated methods and approaches continue to be developed and applied to the nurse rostering problem. Some of these approaches include mixed algorithms and heuristics such as a simulated model augmented by artificial intelligence methods, a shift pattern generating heuristic, and a simulated annealing algorithm. These are just a few of the more advanced techniques applied to the nurse rostering problem (Ernst et al, 2004: 12). In the most recent years, highly complex methods such as tabu searches and genetic algorithms have been applied to the nurse rostering problem.

### **Moz and Pato**

In a 2006 journal article, Moz and Pato examined the nurse re-rostering problem using a constructive heuristic and a genetic algorithm. The goal of their research is to develop a management system for scheduling nurses in Portuguese public hospitals. Using actual data from a Portuguese hospital, the authors applied both a constructive heuristic and a genetic algorithm to the nurse re-rostering problem.



The constructive heuristic entails reassigning all tasks to nurses after a disruption has occurred in the schedule. Two approaches are used in the reassignment procedure. The first approach reorders the tasks to nurses according to the rank order of the nurses in the problem. The second approach randomly reorders the tasks to the nurses. After reordering the tasks, the constructive heuristic assigns each task to a nurse. All constraints are upheld during this procedure. First, the procedure attempts to assign the task to a nurse already scheduled for a different task on the same day. If this does not produce a feasible solution, then a backtracking procedure is used to reassign the task to a different nurse, accounting for the attempts that have already occurred to schedule the task. This procedure is iterated until all unassigned tasks are assigned.

In the genetic algorithm procedure, the first step to re-rostering a disrupted schedule is to identify all sets of tasks and nurses, which the authors refer to as the permutation space (Moz and Pato, 2007:673). Each permutation of the list tasks is grouped with each permutation of the list of nurses. These sets of individual groupings of all permutations are defined as chromosome pairs. These pairs represent all feasible and infeasible solutions to the problem. The pairs (or individuals) are then scored based on their similarity to the original schedule. This score is defined as a fitness value. A genetic algorithm (using selection, crossover, and mutation operators) is then applied to the population of individuals to produce a new population (the next generation). This algorithm is stopped after the maximum number of generations has been reached or the best fitness value has not improved after a defined number of generations.

The performance of the constructive heuristic and the genetic algorithm were measured by examining the number of feasible solutions, the number of optimal

solutions, and the average computational time. Using the constructive heuristic, the randomly assigned nurse and task listings outperformed the rank-ordered nurse and task listing. Out of 67 instances, 67 feasible solutions and 39 optimal solutions were found in an average of 0.75 seconds. Using the genetic algorithm methods produced almost 60 optimal solutions, but the average computational time increased to 19 minutes.

### **Knighton's Model**

In his 2005 doctoral dissertation, Knighton examined a network-based mathematical programming approach to using employee preferences in re-rostering optimal workforce schedules. The goal of this methodology is to respond to disruptions in a workforce schedule, while minimizing the number of deviations to the original schedule. Knighton examines rostering a continuous heterogeneous workforce over a multi-week period. A set of employee shift preferences and management employee weights are used as constraints in the model. The employee shift preferences identify the shifts that each employee prefers to work. The management employee weights identify the rank order that the schedule manager uses to assign work shifts. An employee with a high management employee weight will receive preference for a shift over an employee with a lower weight.

After identifying the constraints to the problem, Knighton uses a network-based linear program to determine the optimal rostered schedule. The problem is formulated as a "minimum cost network-flow, using an arc capacity method" (Knighton, 2005:67). Although the workforce scheduling problem is a binary set-covering problem, and this formulation is not an integer program, the network structure does generally provide

integer solutions. The network-based model is defined by the following set of equations

(Knighton, 2005:72):

$$\sum_i s_{j,k,d} e_i = demand \quad \text{for all } j,k,d \quad (2.1)$$

$$\sum_{j,k} s_{j,k,d} e_i = D_{d,i} \quad \text{for all } i,d \quad (2.2)$$

$$\sum_d D_{d,i} \leq \max number\_of\_shifts \quad \text{for all } j \quad (2.3)$$

$$\sum_d D_{d,i} \geq \min number\_of\_shifts \quad \text{for all } j \quad (2.4)$$

$$0 \leq D_{d,i} \leq 1 \quad \text{for all } i,d \quad (2.5)$$

$$0 \leq \sum_w W_{m,j} \leq \max weekend\_shifts \quad \text{for all } j \quad (2.6)$$

According to Knighton's formulation, " $s_{j,k,d}$  denotes shift number  $j$  requiring skill set  $k$  and on day  $d$ ,  $e_i$  is employee number  $i$ ,  $D_{d,i}$  is the total number of shifts,  $D$ , on day  $d$  for employee  $i$ , and  $E_i$  is the total number of shifts per week,  $E$ , for employee  $I$ "

(Knighton, 2005:69). Therefore, the first constraint requires that the employee demand per shift is satisfied. The second and fifth constraints require that no employee works more than one shift per day. The third and fourth constraints ensure that each employee works at least the required minimum, and not more than the allowable maximum number of shifts each week. The final constraint limits the number of weekend shifts,  $W_{m,j}$ , an employee can work during the scheduling period,  $m$  (Knighton, 2005:72).

Knighton (2005:69) explains his model as follows:

The number of employees needed for each shift node flows from the Demand arc. Each shift node has an edge to each qualified and available employee node. The shift-to-employee arc is capacitated at 1, meaning only one of the required staffing for a shift can be assigned to a single employee. Each employee then flows their daily work assignment to the employees' daily-total-node,  $D$ . This new arc is capacitated at 1, meaning each employee can work only one shift per day. Finally, the employees' daily-total-nodes channel the flow to the employees' weekly-total-node

which contains the capacities to enforce minimum and maximum shifts per week.

By combining multiple weekly workforce scheduling problems, a schedule is constructed for longer time horizons. Figure 2.1 illustrates Knighton's network based workforce scheduling model.

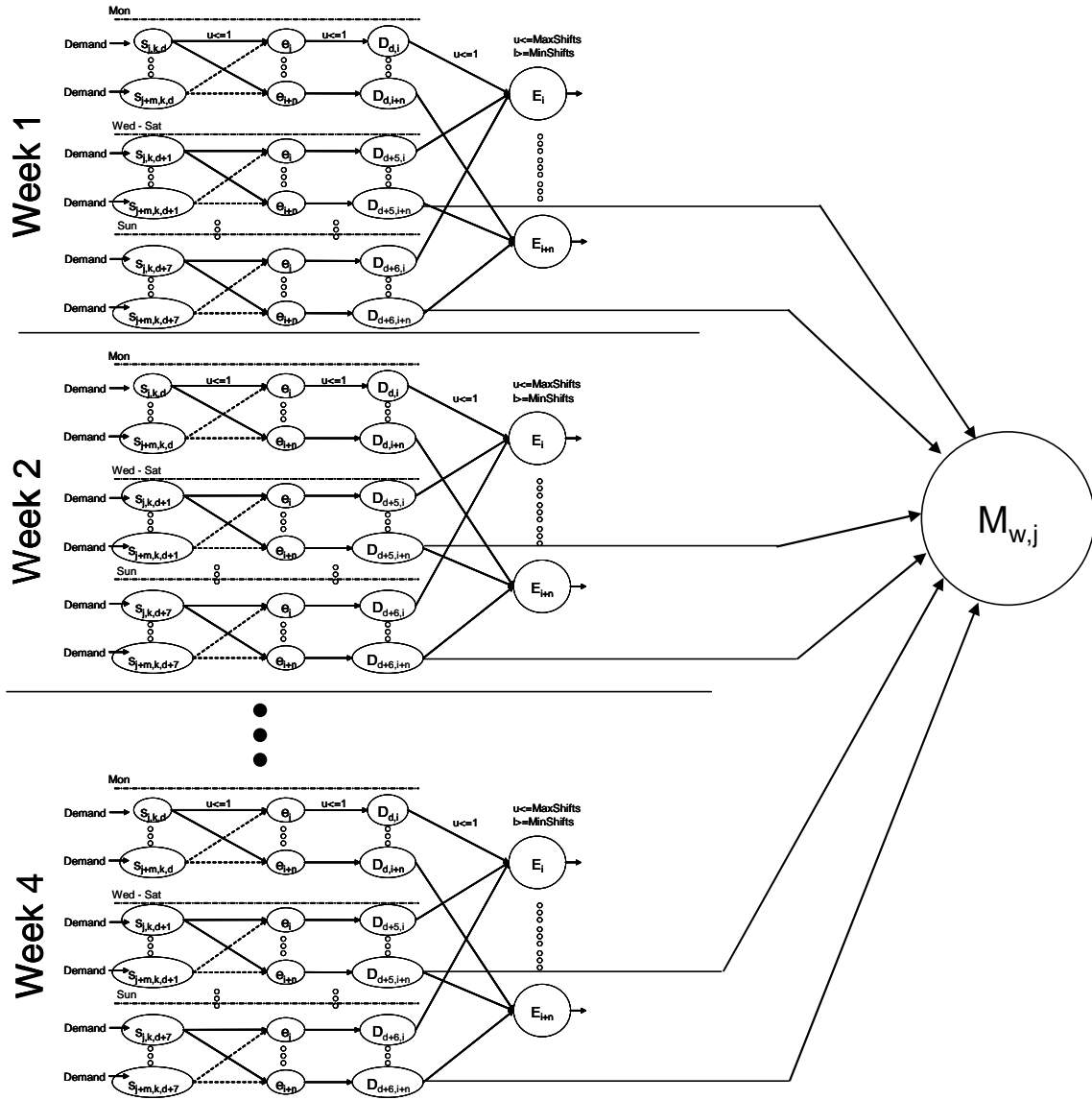


Figure 2.1. Knighton's Multi-Weekly Network Representation of Workforce Scheduling Problem

## **Uncertainty and Robust Scheduling**

Optimization methods provide excellent solutions to workforce scheduling problems. However, in the midst of uncertainty, these solutions may no longer be valid. When optimal schedules are introduced into a real world environment, the likelihood that the schedule will remain valid is very low (Davenport, 1999: 1). Contingencies occur, such as sickness, which preclude employees from being able to perform their duties in accordance with the optimal schedules. When these events, or disruptions, occur, the scheduler must ensure that the schedule remains valid in accordance with industry regulations and local laws.

Within the scheduling environment, there are two approaches for dealing with uncertainty (Davenport, 1999: 3). A scheduler can be proactive in dealing with uncertainty or reactive. The choice between the two will be dependent on the likelihood that a disruption will occur that may render the schedule invalid.

In reactive scheduling, the scheduler waits for a disruption to render the current optimal schedule invalid. Once the schedule is invalid, the scheduler can then apply a re-rostering method to rebuild the schedule. Knighton (2005) developed a network-based mathematical approach and Moz and Pato (2007) developed a genetic algorithm approach. Often times, the goal of re-rostering is to rebuild a new optimal schedule while minimizing the number of deviations to the schedule. A deviation occurs when an employee must change a previously scheduled shift in order to develop a new optimal schedule. Although re-rostering methods produce new optimal schedules, they often result in deviations, which may not be very well accepted by the workforce.

In contrast to reactive scheduling, proactive scheduling attempts to deal with the uncertainty of contingent events ahead of time (Davenport, 1999: 3). Three methods are available for dealing with uncertainty. First, the scheduler can develop robust schedules. A robust schedule is a schedule that can “absorb environmental uncertainties” and still remain valid (Davenport, 1999: 3). Within workforce scheduling, environmental uncertainties are manifested as employee absences. The second option for pro-active scheduling is to develop contingent schedules. Contingent schedules are developed, but not published until a disruption occurs, invalidating the original schedule. The final option is for the scheduler to develop decision theory approaches for responding to the disruption.

Although robust scheduling is an important topic in scheduling theory, it has received little attention in the scheduling literature. Davenport (1999) presents three possible definitions for robust scheduling. First, a robust schedule is one that remains valid under a wide array of disruptions. Second, a robust schedule is one that is still valid, even when the underlying assumptions may be violated. Finally, a robust schedule is one that is able to satisfy demand requirements in an uncertain environment.

### **Robust Scheduling in Airline Crew Scheduling**

The aforementioned measures of robustness have been applied to areas of manufacturing scheduling (Davenport, 1999: 3), but little work has been done in the area of workforce scheduling. One of the few recent applications is in the airline industry. Shebalov and Klabjan (2006) examined a common problem facing many of today's commercial passenger airlines: How can an airline best schedule aircrews to meet the

demand of specified aircraft routing decisions? In this crew pairing scheduling problem, the objective is to minimize the cost associated with crew scheduling while meeting aircraft routing demand. Although an optimal or near-optimal solution exists for each of these large-scale integer programs, the solution may not be robust against deviations such as delayed flights, sick crews, and other unexpected circumstances. Deviations may cause unexpected operational cost increases of 4% in larger fleets and 8% in smaller fleets. Therefore, the goal of Shebalov and Klabjan's research was to solve a modified integer program that produced "robust crew schedules"--crew schedules that could be modified and adapted based on deviations within the scheduled plan, still meeting scheduled aircraft routing demand but with minimal cost increases.

The traditional crew scheduling problem is modified by adding a second objective of maximizing the number of move-up crews--"crews that can potentially be swapped in operations." In the traditional airline crew pairing problem, the objective is to identify the minimum cost pairings that cover all required routes. The crew pairing model with move-up crew count not only identifies an optimal or near optimal solution, but it also identifies changes to the crew pairings that still provide a feasible solution. In order for a move-up crew to be feasible, it must be available to fly a deviated flight (meet crew rest requirements and positioned at the same crew base) and have the same number of days remaining until the end of the assigned pairing (to prevent disrupting other scheduled flights). The new objective function is to maximize the number of move-up crews associated with each flight assigned to a leg. This objective function supersedes the original objective function of minimizing crew costs. Therefore, the crew pairing problem is first solved with the objective of minimizing crew costs. Then, the problem is

resolved with the new objective of maximizing the number of move-up crews, but with a new constraint on the crew cost (as defined by the solution to the first problem). The flexibility in the crew cost constraint is defined by the operations manager's willingness to increase cost in order to increase the number of move up crews.

The case for robust aircrew scheduling is highlighted by the rising operating costs of the airline industry. Major United States domestic air carriers budget upwards of \$1.4 billion annually towards crew costs (Schaefer et al, 2005: 340). In spite of these high crew costs, disruptions in aircrew schedules continue to increase. The Air Transport Association reported that the average number of delays greater than fifteen minutes increased from 1,416 in 1997 to 2,149 in 1999 (Schaefer et al, 2005: 340). During the same period, the Federal Aviation Administration reported a 58% increase in delays and a 68% increase in flight cancellations (Schaefer et al, 2005: 340). These alarming increases have helped move robust aircrew scheduling to the forefront of the aircrew scheduling literature.

## **Conclusion**

Although the data to support robust scheduling in the nursing industry is lacking, the problem is nonetheless important. As highlighted in the introduction to this thesis, absenteeism in the healthcare industry is a major factor in workforce scheduling. One estimate shows that up to 4% of United Kingdom's National Health System's budget may have been lost to absenteeism (Ritchie et al, 1999: 702). Therefore, it is time to focus attention on developing robust scheduling models within the workforce scheduling academia.



Although the literature surrounding the nurse rostering problem and robust scheduling is limited, the framework exists for developing a robust continuous heterogeneous workforce scheduling model. Knighton's network-based linear programming model is instrumental in building workforce schedules in continuous heterogeneous operations. Shebalov and Klabjan provide insight into building robust service schedules. By combining the core concepts from Knighton's network based linear programming model and Shebalov and Klabjan's robust aircrew scheduling model, potential robust workforce scheduling models are developed in Chapter 3. Schedules are constructed using these models and analyzed in Chapter 4.

### **III. Methodology**

#### **Introduction**

The purpose of this chapter is to present the scheduling methodology used to develop models for producing robust solutions to the nurse rostering problem. First, the nurse rostering problem approached in this thesis is outlined. Second, the model is developed using an integer based mathematical program. The constraints for each model are identified, as well as the objective for each model. This chapter concludes with a presentation of the five models developed and the method used to analyze each model. The analysis is presented in Chapter 4 of this thesis.

#### **Case Study**

Although the size of workforces across nursing care facilities varies, the workforce scheduling problem takes on a standard form. Nurses with varying skill sets must be scheduled to cover defined shifts over a continuous timeline (24 hours, 7 days a week). This research examines a private nursing home located in the state of Maine. This facility was chosen as it fits well into the scope of the nurse rostering problem.

The nursing home provides a staff of trained nurses to care for elderly patients. The nurses assist with the general activities of helping the residents with daily activities. They also provide medical care to the patients, as needed. State law requires that at least one licensed nurse be on staff at all times (Oliver, 2007). A licensed nurse is defined as either a registered nurse or a licensed practical nurse.

Other staffing requirements are also defined by the State of Maine. Between the hours of 0700 and 1500 (day shift, D), state law requires that at a minimum of one nurse must be on duty for every five beds in the facility (Oliver, 2007). Between the hours of 1500 and 2300 (evening shift, E), a minimum of one nurse must be on duty for every ten beds. Finally, between the hours of 2300 and 0700 (night shift, N), a minimum of one nurse must be on duty for every 15 beds.

The particular facility examined in this thesis has 20 beds. Therefore, at least four nurses must be available during day shift, at least two nurses must be available during evening shift, and at least two nurses must be available during night shift. Finally, at least one licensed nurse must be on staff during each of these shifts. Eight of the 20 assigned nurses are licensed nurses.

The nursing home also dictates a few other workforce constraints. First, each nurse should be scheduled for a minimum of four shifts per week, but not more than six shifts per week. Furthermore, each nurse will not be scheduled for more than two weekend shifts during a two week period. Finally, each nurse must have a minimum of 8 hours of rest following any work shift of 16 hours. All schedules are constructed for a two week period.

## **Model Construction**

An integer based mathematical program is used as the basis for modeling the workforce scheduling problem. This approach is based on the network-flow based mathematical program developed by Knighton in his doctoral dissertation (2005). The network-flow approach provides a solid framework for modeling the nurse rostering

problem. By changing the objective function and constraint sets, the model can provide a robust solution, rather than an optimal solution. Figure 3.1 shows the network based approach used by Knighton to solve the workforce scheduling problem.

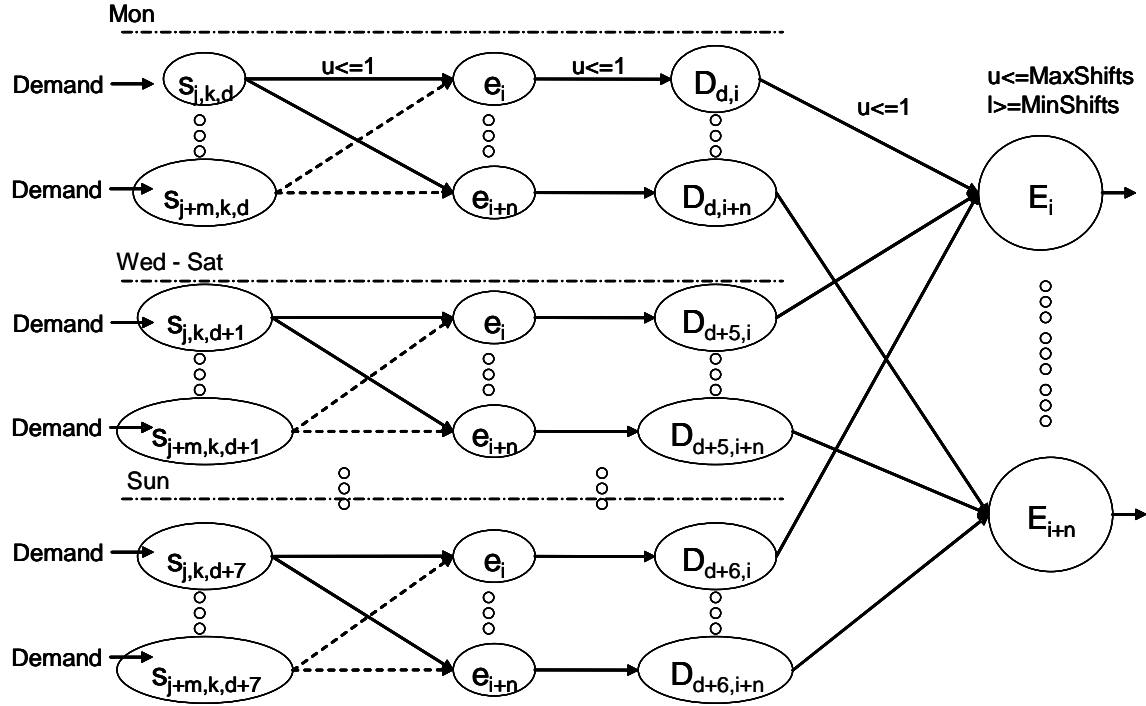


Figure 3.1. Knighton's Network-Based Mathematical Model

On any given shift  $j$  skill set  $k$  is required per day  $d$ . The first skill set is for a licensed nurse. There must be one licensed nurse on staff each shift. The second skill set is for any nurse. For this case, a value of 1 indicates that any nurse can fill the shift demand. A value of 2 indicates that only a licensed nurse can fill the shift demand. Therefore, the demand per shift per day per skill set is represented by  $s_{j,k,d}$ . The shift variable  $j$  has values of 1, 2, and 3 which correspond to day shift, evening shift, and night shift. Each employee  $e_i$  is available to fill shift demand as long as they possess the appropriate skill set  $k$  (where  $i$  represents employee 1, 2, ...,  $n=20$ ).

The total number of shifts an employee is scheduled for each day flows into the total shift node  $D_{d,i}$ . For this case, each employee is limited to working one shift per day. The total number of shifts per day flows into the total number shifts per week node  $E_i$ . Each employee must be scheduled for a minimum of 4 shifts, but may not be scheduled for more than 6 shifts per week. Furthermore, each employee is limited to working no more than two weekend shifts in a two week period. Equations 2.2 through 2.6 in Chapter 2 describe the model above. The primary changes to Knighton's construct occur in equation 2.1. The variable  $s_{j,k,d}$  and  $e_i$  are combined into a single binary variable  $e_{i,j,k,d}$ . This variable defines whether an employee  $i$  is assigned to shift  $j$  under skill set  $k$  on day  $d$ . Furthermore, the sum of this variable for all employees must be greater than or equal to demand, instead of equal to. This allows for more robust shifts if employees are available for duty. The modified form of equation 2.1 is presented below as equation 3.1:

$$\sum_i e_{i,j,k,d} \geq demand \quad \text{for all } j,k,d, \text{ where } e_{i,j,k,d} \text{ is binary.} \quad (3.1)$$

### **Availability Constraint**

An optimal schedule seeks to solve the model above by defining the cost associated with each employee being assigned to each demand shift. The cost can be defined in dollars or employee preference (Knighton, 2005). In contrast to an optimal schedule, the critical objective of building a robust schedule is maximizing the number of employees available to work each day. Therefore, there may not be a unique optimal solution. Rather, there may be a set of feasible solutions that maximizes the number of shifts covered with additional employees. The solution set will be any solution that

maximizes the number of employees available to work each day, while meeting the original workforce scheduling demands.

### **Assumptions**

Some key assumptions to each model are that each employee can work back-to-back shifts. However, the shifts can not occur on the same day and can not violate the rule that each nurse must have 8 hours of rest following 16 hours on duty. For example, a nurse may be scheduled to work night shift on a Tuesday and may also be scheduled to work day shift on Wednesday. Although this may be uncommon in the real world, it is feasible and it simplifies the number of constraints used to develop the model.

### **Model 1: Basic Work Schedule Model**

The first model developed is the basic work schedule model. The purpose of this model is to minimally satisfy all constraints. This model provides a baseline for the average number of disruptions that the nursing home schedule will be able to encounter before the schedule becomes invalid.

In this model, all 20 nurses are scheduled for the minimum of 4 shifts per week and no more than 2 weekend shifts. A total of 160 shifts must be scheduled. State law only requires 112 shifts to be scheduled during each 2-week scheduling period, based on the 20 beds assigned to the nursing home. Therefore, the additional 48 shifts are evenly distributed across the week days and weekend days to ensure that each day has a balanced number of nurses scheduled. On each weekday, five nurses and one licensed nurse are scheduled for the day shift (D), two nurses and one licensed nurse are scheduled

for the evening shift (E), and one nurse and one licensed nurse are scheduled for the night shift (N). On each weekend day, four nurses and one licensed nurse are scheduled for the day shift, two nurses and one licensed nurse are scheduled for the evening shift, and one nurse and one licensed nurse are scheduled for the night shift. Table 3.1 shows the shift demand for Model 1. In this model, no attention is given to the licensed nurses (defined as  $k = 2$ ) to ensure that they are evenly distributed across shifts. The objective function for this model is to minimize the total number of shifts scheduled, while meeting all other constraints. The objective function is shown below as equation 3.2:

$$\min \sum_i e_{i,j,k,d} \quad \text{for all } i,j,k,d \quad (3.2)$$

TABLE 3.1 Shift Demand															
	For d = 1...5, 8...12						For d = 6, 7, 13, 14						Nurses SchedMin ShiftsSch Shifts		
	s <sub>1,1,d</sub>	s <sub>1,2,d</sub>	s <sub>2,1,d</sub>	s <sub>2,2,d</sub>	s <sub>3,1,d</sub>	s <sub>3,2,d</sub>	s <sub>1,1,d</sub>	s <sub>1,2,d</sub>	s <sub>2,1,d</sub>	s <sub>2,2,d</sub>	s <sub>3,1,d</sub>	s <sub>3,2,d</sub>			
Minimum	3	1	1	1	1	1	3	1	1	1	1	1	112		
Model 1	5	1	2	1	1	1	4	1	2	1	1	1	20	150	160
Model 2	5	1	3	1	3	1	3	2	1	2	1	1	20	180	180
Model 3	4	2	2	2	2	2	3	2	1	2	1	1	20	180	180
Model 4	3	1	2	1	1	1	3	1	1	1	1	1	16	122	128
Model 5	3	1	2	1	1	1	3	1	1	1	1	1	16	122	128

## Model 2: Strengthened Work Schedule Model

The second model developed is the strengthened work schedule model. The purpose of this model is to increase the robustness of each shift by scheduling additional employees on each shift. This model is the first of the four robust scheduling models examined in this thesis.

In this model, all 20 nurses are scheduled for a minimum of four shifts per week, but no more than five shifts per week. Furthermore, the weekend constraint is relaxed and each nurse may work no more than three weekend shifts during the scheduling period. A total of 180 shifts are scheduled to make the strengthened work schedule model more robust than the basic work schedule model. Because state law only requires 112 shifts to be scheduled during each 2-week scheduling period, the additional 62 shifts are evenly distributed across the week days and weekend days to ensure that each day has a balanced number of nurses scheduled. On each weekday, five nurses and one licensed nurse are scheduled for the day shift, three nurses and one licensed nurse are scheduled for the evening shift, and three nurses and one licensed nurse are scheduled for the night shift. On each weekend day, three nurses and two licensed nurse are scheduled for the day shift, one nurse and two licensed nurse are scheduled for the evening shift, and one nurse and one licensed nurse are scheduled for the night shift. Table 3.1 shows the shift demand for Model 2. In this model, no attention is given to the licensed nurses to ensure that they are evenly distributed across each weekday shift. However, on the weekend day, both day shift and evening shift have an additional licensed nurse scheduled for duty. The objective function for this model is to minimize the number of shifts scheduled while meeting all other constraints.

### **Model 3: Strengthened and Balanced Work Schedule Model**

The third model developed is the strengthened and balanced work schedule model. The purpose of this model is to increase the robustness of each shift by



scheduling additional employees on each shift. This model is the second of the four robust scheduling models examined in this thesis.

This model is very similar to Model 2. All 20 nurses are scheduled for a minimum of four shifts per week, but no more than five shifts per week. Furthermore, the weekend constraint is relaxed and each nurse may work no more than three weekend shifts during the scheduling period. A total of 180 shifts are scheduled to make this model more robust than the basic work schedule model. Because state law only requires 112 shifts to be scheduled during each 2-week scheduling period, the additional 62 shifts are evenly distributed across the week days and weekend days to ensure that each day has a balanced number of nurses scheduled. In contrast to Model 2, the number of licensed nurses scheduled for each shift is increased. On each weekday, four nurses and two licensed nurse are scheduled for the day shift, two nurses and two licensed nurse are scheduled for the evening shift, and two nurses and two licensed nurse are scheduled for the night shift. On each weekend day, three nurses and two licensed nurses are scheduled for the day shift, one nurse and two licensed nurses are scheduled for the evening shift, and one nurse and one licensed nurse are scheduled for the night shift. Table 3.1 shows the shift demand for Model 3. The objective function for this model is to minimize the number of shifts scheduled while meeting all other constraints (see equation 3.2).

#### **Model 4: Reserve Work Schedule Model**

The fourth model developed is the reserve work schedule model. The purpose of this model is to increase the robustness of the work force by only scheduling the minimum number of employees required to meet the minimum shift requirements as

defined by state law. In this case, only 16 employees are needed to meet the minimum shift requirements. The additional four nurses are kept in reserve.

One key assumption in analyzing this model is that a reserve nurse is available to cover any shift that fails to meet the minimum shift requirements due to a disruption, given that the nurse meets the skill requirement of the disrupted shift. Another key assumption is that a reserve nurse is only allowed to work five shifts during the scheduling period, instead of the ten shift per scheduling period regularly scheduled nurses may work. This assumption is presented to offset the potentially higher wage rate or salary due to a reserve nurse's volatile work schedule. A reserve nurse is still subject to all other employee scheduling constraints. In this model, the four reserve nurses are all non-licensed nurses. This model is the third of the four robust scheduling models examined in this thesis.

All 16 nurses are scheduled for a minimum of four shifts per week and may not be scheduled for more than two weekend shifts during the scheduling period. A total of 128 shifts must be scheduled. Because state law only requires 112 shifts to be scheduled during each 2-week scheduling period, the additional 16 shifts are evenly distributed across the week days and weekend days to ensure that each day has a balanced number of nurses scheduled. On each weekday, three nurses and one licensed nurse are scheduled for the day shift, two nurses and one licensed nurse are scheduled for the evening shift, and one nurse and one licensed nurse are scheduled for the night shift. On each weekend day, three nurses and one licensed nurse are scheduled for day shift, one nurse and one licensed nurse are scheduled for the evening shift, and one nurse and one licensed nurse are scheduled for the night shift. Table 3.1 shows the shift demand for Model 4. The

objective function for this model is to minimize the number of shifts scheduled while meeting all other constraints (see equation 3.2).

### **Model 5: Alternate Reserve Work Schedule Model**

The fifth and final model developed is the alternate reserve work schedule model. Much like the reserve work schedule model, the purpose of this model is to increase the robustness of the work force by only scheduling the minimum number of employees required to meet the minimum shift requirements as defined by the state law. The additional four nurses are kept in reserve. Again, one key assumption in analyzing this model is that a reserve nurse is available to cover any shift that fails to meet the minimum shift requirements due to a disruption, given that the nurse meets the skill requirement of the disrupted shift. Also, each reserve nurse may only work five shifts. A reserve nurse is still subject to all other employee scheduling constraints. In this model, three reserve nurses are all non-licensed nurses. The fourth reserve nurse is a licensed nurse who can cover any shift. This model is the last of the four robust scheduling models examined in this thesis.

All 16 nurses are scheduled for a minimum of four shifts per week and may not be scheduled for more than two weekend shifts during the scheduling period. A total of 128 shifts must be scheduled. Because state law only requires 112 shifts to be scheduled during each 2-week scheduling period, the additional 16 shifts are evenly distributed across the week days and weekend days to ensure that each day has a balanced number of nurses scheduled. On each weekday, three nurses and one licensed nurse are scheduled for the day shift, two nurses and one licensed nurse are scheduled for the evening shift,

and one nurse and one licensed nurse are scheduled for the night shift. On each weekend day, three nurses and one licensed nurse are scheduled for the day shift, one nurse and one licensed nurse are scheduled for the evening shift, and one nurse and one licensed nurse are scheduled for the night shift. Table 3.1 shows the shift demand for Model 5. The objective function for this model is to minimize the number of shifts scheduled while meeting all other constraints (see equation 3.2).

### **Disruptions and Analysis**

After building a feasible work schedule for each model, the models are analyzed based on each model's ability to respond to disruptions in the work schedule. This thesis assumes that schedule disruptions are random and do not follow any formal pattern. Therefore, disruptions pairs are randomly generated from two uniform distributions. The first number  $X$  in each disruption pair identifies the employee who is unavailable for work. The second number  $Y$  in each pair identifies the day in the scheduling period that the employee is unavailable to work. For example, a disruption pair of (16,4) indicates that employee 16 is no longer available to work on day 4. If employee 16 is scheduled for duty on day 4, then the schedule is disrupted and evaluated to see if it is still valid. A valid schedule is a schedule that meets the minimum shift requirements as required by state law.

Each model is evaluated based on its response to twenty sets of disruption pairs. Each set contains 100 disruption pairs. Only pairs that affect employees on scheduled duty days are evaluated. If the disruption pair represents an employee on a day off, then

the pair is skipped. All five models are evaluated based on the same 20 sets of disruption pairs to minimize the variance across models.

During evaluation of each work schedule, the first affected shift is disrupted. The schedule is then evaluated to determine if it still meets minimum shift requirements. If the schedule is still valid, then the next disrupted shift is evaluated. This process continues until a shift is disrupted which renders the schedule invalid. After a schedule is invalid, the number of disruptions is recorded. Figure 4.1 in Chapter 4 gives an example of a disruption set and its effect on a valid schedule. After evaluating all 20 sets of disruptions, the data set provides a measure of the robustness of a model to disruptions. That is, each model is then compared based on the average number of disruptions it could receive before becoming invalid.

Although the focus of this research is on building disrupted schedules, the cause of schedule failure is also recorded. A schedule can become invalid for two reasons. First, each shift requires one licensed nurse to be on duty. If a licensed nurse's shift is disrupted and an additional licensed nurse is not scheduled for duty (Models 1, 2, 3, and 4) or a licensed nurse is not available to cover the shift (Model 5), then the schedule is no longer valid. The schedule fails due to a lack of a licensed nurse or skilled employee. The schedule can also fail for another reason. If a schedule is disrupted and the number of scheduled nurses falls below the minimum state requirements, then the schedule is also no longer valid for Models 1, 2, and 3. For Models 4 and 5, the schedule is no longer valid when all available reserve nurses have been used to cover five disrupted shifts each. In this case, the schedule fails due to a shortage of manpower.

## **Conclusion**

The robust workforce scheduling models provide managers with tools to ensure that an adequate workforce is available in the event that a disruption occurs in the workforce schedule. As defined in Chapter 1, a disruption is any event that prevents a scheduled worker from being able to perform his or her duty on a give shift or day. In the next chapter, the case study of building a robust workforce schedules for a nursing home staff is examined and evaluated for robustness.

## **IV. Results and Analysis**

### **Introduction**

The purpose of this chapter is to present the results from the five models developed in Chapter 3. Each model is presented, as well as the schedule developed using the model. Data was gathered by randomly generating disturbances for each schedule. Then, each model was measured by its ability to adequately respond to the disruptions without significant changes to the schedule. By definition, a significant change to the schedule is a change that requires the model to be re-rostered using a mathematical program. The response to disruptions for each model was analyzed using statistical analysis of variance techniques to determine if one schedule is more robust than another schedule. Furthermore, statistical testing of population proportions was used to determine if a schedule was failing for a particular cause more than another schedule. At the end of this chapter, results are presented. The results are discussed in Chapter 5.

### **Model 1**

The first model developed is the basic work schedule model. This model minimally satisfies all constraints. Each employee is scheduled for at most one shift per day and only four shifts per week. Furthermore, each employee may only work two weekend shifts during the two week period. Based on the shift demand requirements identified in Chapter 3, only 112 shifts are required. (A minimum of 4 employees must be scheduled for day-shift, 2 employees for evening shift, and 2 employees for night

shift.) Because all 20 employees must work a minimum of 4 shifts per week, 160 employee shifts must be scheduled. Therefore, the remaining 48 employee shifts were distributed evenly across each day of the week as discussed in Chapter 3. Table 4.1 presents the basic work schedule developed using Model 1.

		Week 1							Week 2						
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Licensed Nurses	1	O	D	D	O	E	O	N	E	D	O	N	O	N	O
	2	D	D	O	O	O	N	D	D	N	E	O	N	O	O
	3	D	E	E	N	O	O	O	O	O	N	D	O	D	N
	4	N	O	D	O	D	D	O	N	E	O	E	O	D	O
	5	D	N	D	O	O	O	D	E	E	O	O	D	D	O
	6	E	O	N	E	D	O	O	D	O	E	O	O	N	E
	7	D	O	O	D	D	O	E	O	O	D	D	E	E	O
	8	D	E	O	O	N	E	O	O	O	D	D	D	O	D
Non-Licensed Nurses	9	N	D	O	D	O	O	D	E	D	O	N	O	O	D
	10	O	N	O	D	D	D	O	N	O	N	E	O	O	D
	11	O	O	O	N	N	N	E	O	D	D	D	N	O	O
	12	O	D	E	O	D	E	O	D	D	O	O	E	E	O
	13	O	N	O	N	O	D	D	D	N	O	D	D	O	O
	14	E	O	D	E	O	D	O	O	D	D	O	E	O	N
	15	E	O	O	D	D	O	D	D	N	O	O	E	D	O
	16	O	O	N	D	N	O	N	N	O	E	D	O	O	D
	17	D	D	D	O	O	D	O	D	O	N	E	O	O	E
	18	N	D	E	E	O	O	O	O	E	O	O	D	E	D
	19	O	O	D	D	E	O	E	O	D	D	O	D	O	E
	20	O	E	N	O	E	E	O	O	O	D	N	D	D	O

## Model 2

The second model developed is the strengthened work schedule model. Each employee is scheduled for at most one shift per day and a minimum of four shifts per week, but no more than five shifts per week. However, the weekend constraint is relaxed to allow any employee to work up to three weekend shifts during the two week period. Most importantly, each weekday shift is strengthened by scheduling two additional



employees per shift. No attention is given as to whether or not the additional employee is a licensed nurse or a non-licensed nurse. Each weekend day is strengthened by scheduling two additional employees on day shift, and one additional employee on evening shift. A total of 180 employee shifts must be scheduled. Table 4.2 presents the strengthened work schedule developed using Model 2.

		Week 1							Week 2						
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Licensed Nurses	1	D	D	N	D	E	O	O	O	O	E	N	N	E	O
	2	E	D	O	N	D	O	D	O	N	O	D	D	E	O
	3	D	D	N	D	O	O	E	D	D	D	O	O	D	D
	4	O	N	D	N	D	D	O	O	O	E	E	D	N	E
	5	O	O	E	D	E	E	D	E	D	O	D	E	O	D
	6	N	E	O	O	N	N	N	N	D	D	E	O	O	N
	7	D	O	E	E	O	E	E	E	E	N	O	N	O	E
	8	E	D	D	O	O	D	D	N	E	O	N	D	D	O
Non-Licensed Nurses	9	O	N	D	N	N	N	O	D	D	N	O	E	O	O
	10	O	E	D	D	O	O	D	O	E	D	D	O	O	E
	11	D	N	O	E	D	D	O	O	O	D	D	D	D	N
	12	E	D	O	O	N	E	O	N	O	E	E	N	O	O
	13	O	O	N	N	O	D	E	N	D	O	D	D	D	O
	14	N	D	O	O	D	O	N	O	N	D	D	O	D	D
	15	N	O	D	D	D	D	O	D	O	D	N	D	E	O
	16	E	E	D	O	E	E	O	E	N	E	N	O	O	D
	17	D	O	E	O	D	N	D	D	D	O	O	E	E	O
	18	O	E	N	E	E	E	O	E	E	N	O	N	O	O
	19	D	O	E	D	N	O	O	D	N	N	O	O	N	O
	20	N	N	O	E	O	O	N	D	O	O	E	E	O	D

### Model 3

The third model developed is the strengthened and balanced work schedule model. This model is much like Model 2, except the additional shift employees are further defined as licensed or non-licensed employees. Each employee is scheduled for at most one shift per day and a minimum of four shifts per week, but no more than five

shifts per week. As with Model 2, the weekend constraint is relaxed to allow any employee to work up to three weekend shifts during the two week period. Most importantly, each weekday shift is strengthened by scheduling one licensed nurse and one non-licensed nurse per shift. Each weekend day is strengthened by scheduling two additional employees on day shift, and one additional employee on evening shift. A total of 180 employee shifts must be scheduled. Table 4.3 presents the second strengthened work schedule developed using Model 3.

**TABLE 4.3. Model 3 Work Schedule**

		Week 1							Week 2						
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Licensed Nurses	1	E	N	O	N	O	E	N	N	E	O	E	D	E	O
	2	N	D	O	E	N	O	E	O	D	N	O	N	N	D
	3	D	E	D	N	E	O	O	D	O	E	E	E	O	E
	4	D	E	E	O	D	N	O	N	E	O	N	D	O	D
	5	O	D	D	D	N	E	O	E	O	N	D	E	D	O
	6	O	O	N	D	D	D	E	O	N	E	D	N	E	O
	7	E	O	N	O	E	D	D	E	N	D	N	O	O	E
	8	N	N	E	E	O	O	D	D	D	D	O	O	D	N
Non-Licensed Nurses	9	D	N	D	O	D	O	O	E	O	N	D	O	O	D
	10	N	E	O	D	D	O	O	D	D	O	O	D	E	O
	11	E	O	E	O	O	D	E	E	D	E	O	O	N	O
	12	D	D	O	N	E	O	O	N	O	N	E	O	D	O
	13	O	D	N	E	D	O	O	O	O	D	N	D	D	O
	14	D	O	E	N	O	D	D	N	E	O	O	E	O	D
	15	E	O	O	O	N	N	D	O	O	D	D	N	O	E
	16	D	D	N	D	O	O	O	D	N	E	O	O	D	O
	17	O	O	D	E	D	O	N	O	D	D	D	D	O	O
	18	O	N	D	D	O	D	D	D	N	O	N	E	O	D
	19	O	D	O	D	E	E	O	O	E	D	E	N	O	N
	20	N	E	D	O	N	O	O	D	D	O	D	D	O	O

## Model 4

The fourth model developed is the reserve work schedule model. In this model, four non-licensed nurses (employees 17 thru 20) are not scheduled for work during the work period. The remaining 16 employees are scheduled in the same manner as Model 1. Each employee is scheduled for at most one shift per day and only four shifts per week. Furthermore, each employee may only work two weekend shifts during the two week period. Based on the shift demand requirements, only 112 shifts are required. Because only 16 employees must work a minimum of 4 shifts per week, only 128 employee shifts must be scheduled. The remaining 16 employee shifts were left unspecified and were scheduled based on the model constraints. Table 4.4 presents the first reserve work schedule developed using Model 4.

		Week 1							Week 2						
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Licensed Nurses	1	O	E	O	D	N	E	O	O	N	N	O	N	O	E
	2	N	N	D	O	E	O	O	O	O	O	E	D	N	D
	3	O	N	N	O	E	O	E	O	E	O	N	E	O	N
	4	D	N	E	O	N	O	O	E	O	E	O	O	D	E
	5	E	O	O	N	O	E	N	O	D	N	N	N	O	O
	6	D	D	O	E	O	O	D	N	O	D	D	O	O	N
	7	O	E	O	N	E	N	O	O	O	D	D	D	E	O
	8	O	D	N	O	D	D	O	D	N	O	O	N	D	O
Non-Licensed Nurses	9	D	O	D	O	D	O	N	D	O	D	E	O	O	D
	10	N	O	D	D	O	O	E	N	O	D	N	O	O	D
	11	D	O	O	O	D	N	D	D	D	N	O	E	O	O
	12	E	E	O	D	O	O	D	O	D	E	O	D	E	O
	13	O	O	E	D	D	O	D	E	D	O	O	D	D	O
	14	O	D	O	E	N	D	O	E	N	O	N	O	O	D
	15	O	D	D	N	O	D	O	D	O	O	D	D	N	O
	16	E	E	N	O	O	D	O	E	E	O	D	O	D	O
	17	O	O	O	O	O	O	O	O	O	O	O	O	O	O
	18	O	O	O	O	O	O	O	O	O	O	O	O	O	O
	19	O	O	O	O	O	O	O	O	O	O	O	O	O	O
	20	O	O	O	O	O	O	O	O	O	O	O	O	O	O

## Model 5

The final model developed is the alternate reserve work schedule model. In this model, three non-licensed nurses (employees 18 thru 20) and one licensed nurse (employee 8) are not scheduled for work during the work period. The remaining 16 employees are scheduled in the same manner as Model 1. Each employee is scheduled for at most one shift per day and only four shifts per week. Furthermore, each employee may only work two weekend shifts during the two week period. Based on the shift demand requirements, only 112 shifts are required. Because only 16 employees must work a minimum of 4 shifts per week, only 128 employee shifts must be scheduled. The remaining 16 employee shifts were left unspecified and scheduled based on the model constraints. Table 4.5 presents the reserve work schedule developed using Model 5.

TABLE 4.5. Model 5 Work Schedule															
		Week 1							Week 2						
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Licensed Nurses	1	N	D	O	D	N	O	O	O	O	E	D	O	D	E
	2	O	D	D	N	O	O	N	D	D	O	O	E	O	E
	3	O	N	D	O	O	N	D	O	E	N	E	N	O	O
	4	D	O	N	O	E	D	O	O	O	D	N	D	O	D
	5	E	N	O	E	N	O	O	E	D	O	O	O	N	N
	6	D	O	O	O	D	E	E	N	N	N	O	D	O	O
	7	O	E	E	O	E	O	D	O	D	D	O	D	E	O
	8	O	O	O	O	O	O	O	O	O	O	O	O	O	O
Non-Licensed Nurses	9	O	E	E	O	D	N	O	D	O	E	O	E	D	O
	10	O	O	E	D	O	D	D	D	E	O	D	D	O	O
	11	D	O	O	O	N	E	D	D	N	O	N	E	O	O
	12	E	E	O	E	D	O	O	O	O	N	N	O	N	D
	13	O	N	D	D	O	O	N	O	O	E	D	N	D	O
	14	O	O	N	E	E	O	E	E	O	D	E	O	O	N
	15	E	D	O	N	O	D	O	O	D	D	D	O	O	D
	16	D	D	D	O	N	O	O	N	O	N	O	O	E	D
	17	N	O	O	D	D	D	O	E	E	O	E	O	D	O
	18	O	O	O	O	O	O	O	O	O	O	O	O	O	O
	19	O	O	O	O	O	O	O	O	O	O	O	O	O	O
	20	O	O	O	O	O	O	O	O	O	O	O	O	O	O

## Disruption Generation

After developing the work schedule, disruptions were randomly generated. A random number was generated from a uniform distribution equal to the number of employees scheduled for duty. A second random number was generated from a uniform distribution equal to the number of days in the period. The two random numbers formed a random pairing that represents a potential disruption to the schedule. For example, a random pairing of (16,4) indicates that employee 16 is unavailable for duty on day 4. If employee 16 was originally scheduled for duty on day 4, then this nurse was removed from the schedule. The schedule was then reevaluated to determine if it still met the minimum shift requirements for the disrupted shift. If the schedule was still valid, then another disruption was generated and the schedule was reevaluated. If the schedule was no longer valid, then the total number of disruptions prior to the invalidating disruption was recorded. Each schedule was evaluated based on its response to twenty sets of 100 random disruptions.

Figure 4.1 shows an example of a disruption set and its effect on a valid schedule. The first disruption pair (highlighted by a blue solid line) removes employee 16 from working a day shift on day 4. The schedule remains valid because there is still a sufficient number of employees scheduled for the day shift on day 4. (Remember, state law requires at least four nurses on duty during the day shift, at least two nurses on duty during the evening shift, and at least two nurses on duty during the night shift.) Furthermore, there is at least one licensed nurse still scheduled for duty. The second disruption pair (highlighted by a dashed green line) removes employee 1 from working a night shift on day 7. The schedule is now invalid. There is no longer at least two nurses

scheduled for duty on the night shift. Furthermore, employee 1 was the only licensed nurse scheduled for the night shift on day 7. Therefore, the schedule is invalid due to insufficient manpower and the lack of a licensed nurse.

Model 3 Work Schedule															X	Y	Shift
															16	4	D
															1	7	N
															6	3	N
															5	7	O
															2	13	N
															7	12	O
															1	7	N
															6	3	N
															5	7	O
															2	13	N
															7	12	O

Figure 4.1: Disruption Set's Effect on Model 3 Work Schedule

## Model Responses

Each model was evaluated based on its ability to remain a valid schedule after responding to 20 sets of random disruptions. Table 4.6 outlines the average number of disruptions each model could handle before becoming invalid. The percentage of schedules that failed for skill (lack of a licensed nurse) and manpower are also displayed. A description and analysis of the cause failures will be presented at the end of this chapter. All statistical analysis is conducted with a type I error ( $\alpha_e$ ) of 0.05.

<b>TABLE 4.6. Model Disruption Response</b>						
<b>Model</b>	<b>Sample Size</b>	<b>Avg Disruptions</b>	<b>Min Disruption</b>	<b>Max Disruption</b>	<b>Skill Failure %</b>	<b>Manpower Failure %</b>
<b>1</b>	<b>20</b>	<b>1.37</b>	<b>0</b>	<b>4</b>	<b>100%</b>	<b>15%</b>
<b>2</b>	<b>20</b>	<b>9.35</b>	<b>0</b>	<b>24</b>	<b>60%</b>	<b>50%</b>
<b>3</b>	<b>20</b>	<b>7.60</b>	<b>1</b>	<b>25</b>	<b>60%</b>	<b>85%</b>
<b>4</b>	<b>20</b>	<b>1.00</b>	<b>0</b>	<b>7</b>	<b>100%</b>	<b>0%</b>
<b>5</b>	<b>20</b>	<b>18.20</b>	<b>2</b>	<b>29</b>	<b>85%</b>	<b>20%</b>

### Analysis of Disruptions

Initially analysis of variance (ANOVA) was used to analyze the disruption data. However, the data as a whole failed to meet the underlying assumption of normality. A goodness of fit test was performed on the residual data using a normal distribution. A Wilkes-Shapiro test was applied and had a significance level of 0.0014. The null and alternative hypotheses are:

$H_0$ : The probability distribution is normally distributed

$H_a$ : The underlying probability distribution is not normally distributed

Because significance level does not meet the Type I error level defined above, the disruption data is not normally distributed and analysis of variance cannot be used to analyze the data.

The Friedman  $F_r$ -Test was performed to test if the underlying disruption distribution for each model was the same. The null and alternative hypotheses are:

$H_0$ : The populations of disruptions are identically distributed for all five models

H<sub>a</sub>: At least two of the models have probability distributions of disruptions that differ in location; that is, *at least one* model can absorb more disruptions than the remaining four models.

The Friedman  $F_r$ -statistic is based on the rank sums for each treatment and is defined as

$$F_r = \frac{12}{bp(p+1)} \sum R_j^2 - 3b(p+1)$$

where  $b$  is the number of blocks (in this case, samples),  $p$  is the number of treatments (in this case, models), and  $R_j$  is the  $j$ th rank sum (McClave et al, 2005: 1104). The rank sum is determined by comparing the number of disruptions accepted by the schedules developed using each model. When the number of disruptions between models is the same, the rank sum assigned to each model is the average of the resulting ranks if the models were ranked differently. Table 4.7 shows the disruption data and the rank associated for each model. The Friedman  $F_r$ -statistic is 48.41 and is greater than the  $X^2_{0.05}$  value of 9.49. Therefore, the null hypothesis is rejected and the models likely have probability distributions of disruptions that are in different locations. Specifically, at least one of the five models can absorb more disruptions than the remaining four models.



**TABLE 4.7. Model Disruption Response and Ranks**

Sample	Disruptions					Rank				
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
1	2	12	15	7	16	1	3	4	2	5
2	0	0	4	1	24	1.5	1.5	4	3	5
3	2	2	1	4	22	2.5	2.5	1	4	5
4	0	7	2	0	20	1.5	4	3	1.5	5
5	0	14	3	1	25	1	4	3	2	5
6	0	15	9	2	23	1	4	3	2	5
7	0	6	6	0	12	1.5	3.5	3.5	1.5	5
8	3	3	9	0	15	2.5	2.5	4	1	5
9	4	17	10	2	23	2	4	3	1	5
10	0	0	6	1	22	1.5	1.5	4	3	5
11	2	24	8	2	17	1.5	5	3	1.5	4
12	1	0	9	0	7	3	1.5	5	1.5	4
13	0	9	25	0	15	1.5	3	5	1.5	4
14	2	15	1	0	20	3	4	2	1	5
15	4	5	8	0	2	3	4	5	1	2
16	3	17	1	0	18	3	4	2	1	5
17	0	9	14	0	17	1.5	3	4	1.5	5
18	0	12	14	0	20	1.5	3	4	1.5	5
19	4	7	6	0	17	2	4	3	1	5
20	1	13	1	0	29	2.5	4	2.5	1	5
$R_j$					=	38.5	66	68	33.5	94

The data was then analyzed using the Wilcoxon Rank Sum Test for independent samples. The Wilcoxon Rank Sum Test is nonparametric test that allows statistical comparison between samples regardless of the sample distribution. Much like the Friedman  $F_r$ -Test, the Wilcoxon Test determines if two samples are identical by examining the rank sum of the values within each sample. The null and alternative hypotheses are:

$H_0$ : The disruption probability distribution  $D_A$  for Model A is identical to the disruption probability distribution  $D_B$  for Model B

$H_a$ : The disruption probability distribution  $D_A$  is shifted to the right of the disruption probability distribution  $D_B$ ; that is, Model A can absorb more disruptions than Model B

One-tailed Wilcoxon Rank Sum Tests were performed between each of the five models to determine which models could respond to a higher number of disruptions. The results are displayed in Table 4.8.

<b>TABLE 4.8. Comparison of Disruption Response Across Models</b>					
	<b>Avg Disruptions</b>	<b>Model 2</b>	<b>Model 3</b>	<b>Model 4</b>	<b>Model 5</b>
<b>Model 1</b>	<b>1.37</b>	<b>&lt;0.0001*</b>	<b>&lt;0.0001*</b>	0.2579	<b>&lt;0.0001*</b>
<b>Model 2</b>	<b>9.35</b>	-	0.3929	<b>&lt;0.0001*</b>	<b>0.0003*</b>
<b>Model 3</b>	<b>7.60</b>	-	-	<b>&lt;0.0001*</b>	<b>&lt;0.0001*</b>
<b>Model 4</b>	<b>1.00</b>	-	-	-	<b>&lt;0.0001*</b>
<b>Model 5</b>	<b>18.20</b>	-	-	-	-

\* Statistically significant difference at 0.05.

### **Analysis of Cause of Failure**

A schedule could fail for two reasons. First, when a disruption occurred to an employee on any given shift, there must be at least one extra employee scheduled in that shift to meet the minimum shift demand. Second, when a disruption removed a licensed nurse from a shift, another licensed nurse must have already been scheduled for that shift to meet the minimum demand of one licensed nurse per shift. If a schedule failed because the number of scheduled employees fell below the minimum shift demand, then

the scheduled failed for "manpower". If a schedule failed because a licensed nurse was no longer scheduled for the disrupted shift, then the schedule failed for "skill". The proportion of schedules that failed for skill are shown in Tables 4.9.

<b>TABLE 4.9. Analysis of Percentage of Schedule Failures Due to Skill</b>					
<b>Model</b>	<b>Skill</b>	<b>Sample Size</b>	<b>Adjusted Proportion</b>	<b>Lower Confidence Interval</b>	<b>Upper Confidence Interval</b>
1	100%	20	0.92	0.81	1.03
2	60%	20	0.58	0.39	0.78
3	60%	20	0.58	0.39	0.78
4	100%	20	0.92	0.81	1.03
5	85%	20	0.79	0.63	0.95

The population proportions were analyzed using comparison of population proportions (McClave et al, 2005: 513). Due to the small sample sizes, the population proportions were adjusted to better represent the population (McClave et al, 2005: 378). A comparison of means was applied to determine the models that had a higher failure rate due to a lack of availability of licensed nurses. The results are summarized in Table 4.10.

<b>TABLE 4.10. Comparison of Percentage of Skill Failures Across Models</b>			
<b>Comparison</b>	<b>Difference</b>	<b>Lower Confidence Interval</b>	<b>Upper Confidence Interval</b>
1 & 2*	0.33*	0.11	0.56
1 & 3*	0.33*	0.11	0.56
1 & 4	0	0.00	0.00
1 & 5	0.13	-0.07	0.32
2 & 3	0	0.00	0.00
2 & 4*	-0.33*	-0.11	-0.56
2 & 5	-0.21	-0.46	0.05
3 & 4*	-0.33*	-0.56	-0.11
3 & 5	-0.21	-0.46	0.05
4 & 5	0.13	-0.07	0.32

\* Statistically significant difference at 0.05.

The test results indicate that Models 1 and 4 had higher rates of failure due to skill than Models 2 and 3. Models 1 and 4 had identical rates of failure, as did Models 2 and 3. Model 5's percentage of skill failures was not different from any of the other models.

The proportion of schedules that failed due to manpower are shown in Table 4.11. Each population proportion was adjusted as noted above. A comparison of means was applied to determine which models had a higher failure rate due to a lack of available employees to meet minimum shift demand. The results are summarized in Table 4.12.

**TABLE 4.11. Analysis of Percentage of Schedule Failures Due to Manpower**

Model	Skill	Sample Size	Adjusted Proportion	Lower Confidence Interval	Upper Confidence Interval
1	15%	20	0.21	0.05	0.37
2	50%	20	0.50	0.30	0.70
3	85%	20	0.79	0.63	0.95
4	0%	20	0.08	-0.03	0.19
5	20%	20	0.25	0.08	0.42

**TABLE 4.12. Comparison of Percentage of Manpower Failures Across Models**

Comparison	Difference	Lower Confidence Interval	Upper Confidence Interval
1 & 2*	-0.29*	-0.55	-0.03
1 & 3*	-0.58*	-0.81	-0.35
1 & 4	0.13	-0.07	0.32
1 & 5	-0.04	-0.28	0.20
2 & 3*	-0.29*	-0.55	-0.03
2 & 4*	0.42*	0.19	0.65
2 & 5	0.25	-0.01	0.51
3 & 4*	0.71*	0.51	0.90
3 & 5*	0.54*	0.30	0.78
4 & 5	-0.17	-0.37	0.04

\* Statistically significant difference at 0.05.

The test results indicate that Model 3 had higher rates of failure due to manpower than any other model. Model 2 had a higher rate of failure due to manpower when compared to Models 1 and 4. None of the rest of the comparisons was statistically significant.

## **Conclusion**

The data presented in this chapter indicates that a schedule developed using Model 5 is the most robust. The benefits and consequences of using this schedule will be discussed in the next chapter.

## **V. Discussion**

### **Comparison of Models**

Each of the five models presented in Chapter 3 was used to develop valid workforce schedules for the nurse rostering problem presented in this thesis. The models were analyzed in Chapter 4 using nonparametric statistical analysis. Based on the analysis, it is possible to choose one model that may be considered more robust than another model. This discussion examines the similarity and differences of each model based on its performance in the statistical analysis. Furthermore, the benefits and consequences of using each model are presented.

All five models presented in this thesis employ the 20 nurses assigned to the nursing home. Therefore, from a salary cost perspective, the five models are equivalent. The primary difference between the models is how the employees are scheduled for duty.

The basic work schedule model developed in Model 1 is the simplest of the models developed. No attention is given to how the employees are scheduled, so long as the minimum shift requirements are satisfied in accordance with state laws and industry regulations. Therefore, each nurse is only scheduled for four days of duty during each week and may only work a maximum of two weekend days during the scheduling period.

Although this model requires 160 nurse shifts to gainfully employ all 20 nurses, the model fails to produce a robust schedule. On average, the basic work schedule can only respond to one disruption during the 2-week scheduling period. Any more disruptions invalidate the schedule and require it to be re-rostered, resulting in scheduling deviations. The basic work schedule model provides one of the least robust solutions

when compared to the other four models. The Wilcoxon Rank Sum test was used to compare Model 1 with the other models. Data analysis indicates that the basic work schedule model can absorb statistically less disruptions than Models 2, 3, and 5. When compared to Model 4, the basic work schedule model proved to be equivalent.

The strengthened work schedule model developed in Model 2 produced one of the three robust work schedules. Particular attention was given to how the nurses were scheduled for duty. This ensured that all three shifts, on any given day, were more robust by increasing the number of workers assigned to each shift. Although no attention was given to ensure additional licensed nurses were assigned to each shift, Model 2 proved to be a viable robust solution.

There are significant drawbacks to using Model 2. First, shift robustness was achieved by scheduling nurses to work for more days than the minimum of four days per week. Some nurses worked five days per week, resulting in a total of 180 scheduled shifts. The basic work schedule model only required 160 scheduled shifts. Furthermore, the weekend constraint was relaxed, allowing nurses to work up to three weekend days during the scheduling period. Therefore, the robustness of the schedule developed using Model 2 appears to be a result of the 20 additional shifts worked by the nurses and a result of relaxing the weekend constraint. However, it is interesting to note the dramatic improvement to the robustness of the schedules developed in Model 2, simply by adding 20 additional shifts and relaxing the weekend constraint. Data analysis indicates that the strengthened work schedule model can absorb more disruptions than Models 1 and 4. Furthermore, it is statistically equivalent to Model 3, and not as robust as Model 5.

The strengthened and balanced work schedule model developed in Model 3 proved to be the second of the three robust solutions developed. This model is very similar to Model 2. The main difference between the two models is that particular attention is given to the assignment of the skilled licensed nurses. This attention proved fruitless as Model 3 failed to perform any different than Model 2. (Interestingly, Model 2 appears to be able to handle a higher average number of disruptions than Model 3, but the Wilcoxon Rank Sum test indicates that there is no statistical difference between the two models.)

The reserve work schedule model developed in Model 4 is the second of the two least robust solutions. It is very similar to the basic work force schedule model except four workers are left unscheduled for the week and are placed in a reserve workforce. None of the four workers are only scheduled for duty until after a disruption occurs, which eliminates one of the scheduled nurses from duty. The primary weakness of Model 4 is that only general nurses are placed in the reserve work force. No licensed nurses are set aside to address disruptions.

One benefit of developing the reserve work schedule model is that it only required 128 shifts to employ all 16 nurses. The basic work schedule model required 160 shifts. The results of the analysis on the two models indicate that they are statistically equivalent in their abilities to respond to schedule disruptions. Therefore, if a manager must choose between the two models, the reserve work schedule model may be the better choice. This model satisfies the minimum shift requirements and only requires 16 nurses. The four remaining nurses could be employed elsewhere, or removed from the workforce.



The alternate reserve work schedule model developed in Model 5 proved to be the most robust of the five models examined in this thesis. Model 5 is identical to the reserve work schedule model except that the reserve work pool is modified. One licensed nurse and three general nurses are placed in the reserve work pool. This single difference dramatically improves the reserve work schedule model's ability to respond to disruptions. After comparing the first four models to Model 5 using the Wilcoxon Rank Sum test, Model 5 is mathematically more robust than the first four. The alternate reserve work schedule model was able to handle an average of 18 disruptions in any scheduling period.

The alternate reserve work schedule model proved to be the superior model in this thesis. First, this model required the least amount of scheduled shifts out of the five models. Only 128 shift schedules are assigned to meet the minimum shift demand. Furthermore, each scheduled nurse only works four shifts per week. Second, the maximum weekend constraint is satisfied. Each scheduled nurse only works two weekend days during the scheduling period. This is not the case for Models 2 and 3. Finally, the reserve work schedule model is able to handle more disruptions than any of the other models.

### **Areas for Improvement**

Future research efforts can greatly improve this thesis effort. First, research should be applied to researching workforce scheduling disruptions. In this thesis, disruptions were assumed to be randomly distributed using a uniform distribution. This required all shifts to be made robust because the scheduler would not know which

employees would be disrupted from being able to work. However, improved disruption modeling would allow a scheduler to develop an improved model that only adds robustness to the shifts that have employees that are likely to be disrupted from the work schedule. Therefore, less additional shifts would be required to build a robust work schedule. This could result in a smaller workforce and decreased employment costs.

Another area of improvement is the mathematical program designed to construct the robust models. A simple integer-based mathematical program was used in this thesis. Furthermore, only the basic constraints were included in the model: minimum shift demand, weekend constraints, and skill requirements. Additional work could be accomplished to include employee and manager preferences. This would improve the validity of the model as real-world managers consider seniority and worker preference before building schedules. Furthermore, future research should consider improving the work-rest cycles used in this model. The basic rule used in this thesis is that a nurse may only work one shift per day and no more than two consecutive shifts. However, this did allow nurses to be scheduled for night shift on Day 1, day shift on Day 2, and evening shift on Day 3. Although this satisfies the above constraint, it may be exhausting for a nurse to work three different shifts on three different days.

### **Future Applications**

Currently, the literature regarding robust scheduling is scarce. There are limited applications in airline crew scheduling (Shebalov et al, 2006) and manufacturing flow scheduling in job shops (Davenport, 1999). The concept of robust scheduling is one that most managers would appreciate. Disruptions to workforces and production operations

are inevitable. The ability to respond to disruptions without deviating from an active schedule could provide an organization with a competitive advantage in the service industry or a production operation.

One area to consider applying robust scheduling theory is aircraft scheduling in the United States Air Force. An aircraft is scheduled for flight the week prior to the actual sortie. If the aircraft is broken and unavailable to fly on the day of the sortie, the squadron can add another "spare" aircraft to the schedule. However, if there are not a sufficient number of spares on the schedule, then the squadron must take a deviation if they want to add another aircraft to the schedule. Robust scheduling theory could improve the method that USAF flying units use to schedule flying aircraft. Most importantly, it may identify ways to improve selecting spare aircraft, reducing the number of deviations associated with adding additional aircraft to the schedule.

## **Conclusion**

Disruptions impacting workforce schedules can be costly. Although disruptions can not be eliminated, workforce schedules can be improved to be more responsive to disruptions. This thesis examined five workforce scheduling models designed for a nurse rostering problem and measured their robustness to schedule disruptions. Nonparametric statistical analysis indicated that must be applied to the correct skill sets in order to produce robust workforce schedules. Furthermore, workforce managers can consider leaving a portion of the workforce unscheduled (or in reserve) to accommodate schedule disruptions.

## Appendix A: Model Construction Using Premium Solver

Frontline Systems Premium Solver Platform (version 6.5) for use with Microsoft Excel was used to program and solve the integer-based mathematical models developed in Chapter 3. Below is an outline of the construction of the model within Microsoft Excel.

### Decision Variables:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1		S1,1,1	S1,2,1	S2,1,1	S2,2,1	S3,1,1	S3,2,1	S1,1,2	S1,2,2	S2,1,2	S2,2,2	S3,1,2	S3,2,2	S1,1,3	S1,2,3	S2,1,3	S2,2,3	S3,1,3	S3,2,3	S1,1,4	S1,2,4	S2,1,4	S2,2,4	S3,1,4
2	1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
3	2	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	3	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
5	4	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
6	5	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
7	6	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0
8	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
9	8	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
10																								
11		S1,1,1	S2,1,1	S3,1,1	S1,1,2	S2,1,2	S3,1,2	S1,1,3	S2,1,3	S3,1,3	S1,1,4	S2,1,4	S3,1,4	S1,1,5	S2,1,5	S3,1,5	S1,1,6	S2,1,6	S3,1,6	S1,1,7	S2,1,7	S3,1,7	S1,1,8	S2,1,8
12	9	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	1
13	10	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0
14	11	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	1	0	0	0
15	12	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0
16	13	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0
17	14	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0
18	15	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0	1	0
19	16	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	1	0	0	0
20	17	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
21	18	0	0	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
22	19	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0
23	20	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0

Figure A.1. Excel Screenshot of Decision Variables

For Employees 1 thru 8:

8 Employees can fill 3 possible shifts with 2 possible skills on 14 possible days

672 variables assigned to cells B2:CG9

B2:  $e_{1,1,1,1} \rightarrow$  Employee 7 assigned to shift 1 as skill set 1 on day 1

S8:  $e_{7,3,1,3} \rightarrow$  Employee 7 assigned to shift 3 as skill set 2 on day 3

For Employees 9 thru 20:

12 Employees can fill 3 possible shifts with 1 possible skill on 14 possible days

504 variables assigned to cells B12:AQ23

D15:  $e_{12,3,1,1} \rightarrow$  Employee 12 assigned to shift 3 as skill set 1 on day 1

S22:  $e_{19,3,1,6} \rightarrow$  Employee 19 assigned to shift 3 as skill set 1 on day 6

Constraint:

B2:CG9 is binary

B12:AQ23 is binary

## Individual Shift and Total Shift Constraints

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
25		S1,1,1	S1,2,1	S2,1,1	S2,2,1	S3,1,1	S3,2,1	S1,1,2	S1,2,2	S2,1,2	S2,2,2	S3,1,2	S3,2,2	S1,1,3	S1,2,3	S2,1,3	S2,2,3	S3,1,3	S3,2,3	S1,1,4	S1,2,4	S2,1,4	S2,2,4	S3,1,4
26	Employee																							
27	Shifts	5	1	2	1	2	1	5	1	2	1	2	1	5	1	2	1	2	1	5	1	2	1	2
28	Shift	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=
29	Demand	5	1	2	1	1	1	5	1	2	1	1	1	5	1	2	1	1	1	5	1	2	1	1
30	Total																							
31	Demand	160																						

Figure A.2. Excel Screenshot of Shift Constraints

The sums of employees assigned to shift  $j$ , with skill set  $k$ , on day  $d$  are assigned to cells B26:CG26.

**B25 = SUM(B2:B9,B12:B23)** → sum of employees assigned to shift 1 with skill set 1 on day 1.

**C25 = SUM(C2:C9)** → sum of employees assigned to shift 1 with skill set 2 on day 1.  
(Note: Employees 9 thru 12 are not included in the summation for C25 because employees 9 thru 12 cannot be assigned to skill set 2.)

Constraints:

Minimum shift demand: B26:CG26  $\geq$  B28:CG28

**B30 = SUM(B26:CG26)**

Total sum of all employees assigned to all shifts for all skill sets on all days

## Maximum Daily Shift Constraint

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD
32	Shifts	1	2	3	4	5	6	7	8	9	10	11	12	13	14		1	2	3	4	5	6	7	8	9	10	11	12	13	14
33	per Day	1	0	1	1	0	1	0	1	1	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
34		2	1	1	0	0	0	1	1	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35		3	1	1	1	1	0	0	0	0	0	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
36		4	1	0	1	0	1	1	0	1	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
37		5	1	1	1	0	0	0	1	1	1	0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
38		6	1	0	1	1	1	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
39		7	1	0	0	1	1	0	1	0	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
40		8	1	1	0	0	1	1	0	0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
41		9	1	1	0	1	0	0	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
42		10	0	1	0	1	1	1	0	1	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
43		11	0	0	0	1	1	1	1	0	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
44		12	0	1	1	0	1	1	0	1	1	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
45		13	0	1	0	1	0	1	1	1	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1
46		14	1	0	1	1	1	0	1	0	0	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
47		15	1	0	0	1	1	0	1	1	1	0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
48		16	0	0	1	1	1	1	0	1	1	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
49		17	1	1	1	0	0	1	0	1	0	1	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
50		18	1	1	1	1	0	0	0	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
51		19	0	0	1	1	1	0	1	0	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
52		20	0	1	1	0	1	1	0	0	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure A.3. Excel Screenshot of Maximum Daily Shift Constraint

The sums of all shifts for employee  $i$  on day  $d$  are assigned to cells B33:O52.

For employees 1 thru 8 assigned to cells B33:O40:

**B33 = SUM(B2:G2)**

For employees 8 thru 20 assigned to cells B41:O52:

$$B41 = \text{SUM}(B12:D12)$$

Constraint:

Maximum shifts per day:  $B33:O52 \leq Q33:AD52$

### Weekly Shift and Weekend Constraints

	A	B	C	D	E	F	G	H	I	J	K
56	Shifts per Week								Weekends Shifts		
57		Week1	Week2								
58	1	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
59	2	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
60	3	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
61	4	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
62	5	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
63	6	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
64	7	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
65	8	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
66	9	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
67	10	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
68	11	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
69	12	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
70	13	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
71	14	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
72	15	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
73	16	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
74	17	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
75	18	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
76	19	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2
77	20	4	4	$\geq$	4	$\leq$	5		2	$\leq$	2

Figure A.4. Excel Screenshot of Weekly and Weekend Shift Constraints

#### Week 1:

For employees 1 thru 20 assigned to cells B58:B77:

$$B58 = B33 + C33 + D33 + E33 + F33 + G33 + H33$$

Constraints:

Minimum shifts per week:  $B58:B77 \geq E58:E77$

Maximum shifts per week:  $B58:B77 \leq G58:G77$

#### Week 2:

For employees 1 thru 20 assigned to cells C58:C77:

$$C58 = I33 + J33 + K33 + L33 + M33 + N33 + O33$$

Constraints:

Minimum shifts per week:  $C58:C77 \geq E58:E77$

Maximum shifts per week:  $C58:C77 \leq G58:G77$

Sum of Weekend Shifts:

For employees 1 thru 20 assigned to cells I58:I77:

$$I58 = G33 + H33 + N33 + O33$$

Constraint:

Maximum weekend shifts:  $I58:I77 \leq K58:K77$

### Frontline Systems Premium Solver

The following screenshots show the construction of the model within Premium Solver. Figure A.5. displays the decision variable dialogue box. Figure A.6 displays the constraint dialogue box.

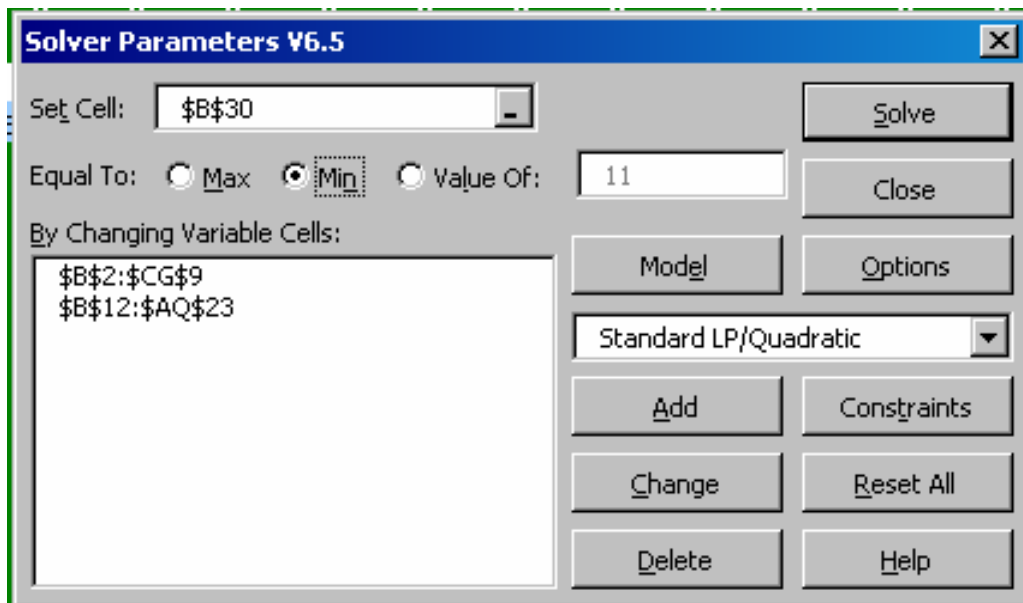


Figure A.5. Excel Screenshot of Decision Variables Dialogue Box

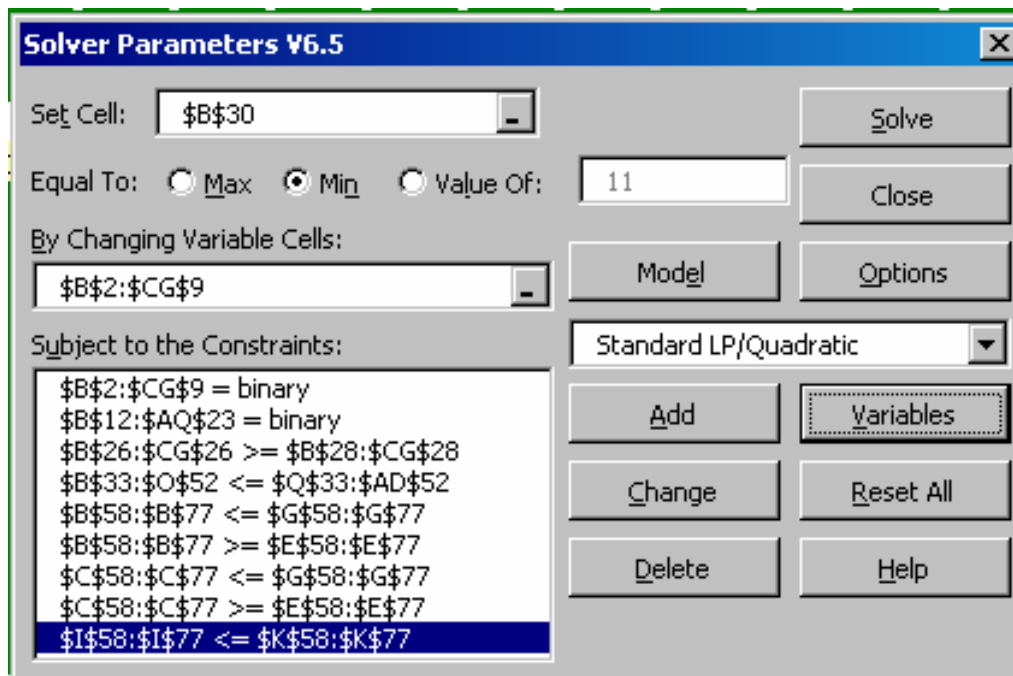


Figure A.6. Excel Screenshot of Constraint Dialogue Box

## Schedule Development

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
82		1	2	3	4	5	6	7	8	9	10	11	12	13	14
83	1	O	D	D	O	E	O	N	E	D	O	N	O	N	O
84	2	D	D	O	O	O	N	D	D	N	E	O	N	O	O
85	3	D	E	E	N	O	O	O	O	O	N	D	O	D	N
86	4	N	O	D	O	D	D	O	N	E	O	E	O	D	O
87	5	D	N	D	O	O	O	D	E	E	O	O	D	D	O
88	6	E	O	N	E	D	O	O	D	O	E	O	O	N	E
89	7	D	O	O	D	D	O	E	O	O	D	D	E	E	O
90	8	D	E	O	O	N	E	O	O	O	D	D	D	O	D
91	9	N	D	O	D	O	O	D	E	D	O	N	O	O	D
92	10	O	N	O	D	D	D	O	N	O	N	E	O	O	D
93	11	O	O	O	N	N	N	E	O	D	D	D	N	O	O
94	12	O	D	E	O	D	E	O	D	D	O	O	E	E	O
95	13	O	N	O	N	O	D	D	D	N	O	D	D	O	O
96	14	E	O	D	E	O	D	O	O	D	D	O	E	O	N
97	15	E	O	O	D	D	O	D	D	N	O	O	E	D	O
98	16	O	O	N	D	N	O	N	N	O	E	D	O	O	D
99	17	D	D	D	O	O	D	O	D	O	N	E	O	O	E
100	18	N	D	E	E	O	O	O	O	E	O	O	D	E	D
101	19	O	O	D	D	E	O	E	O	D	D	O	D	O	E
102	20	O	E	N	O	E	E	O	O	O	D	N	D	D	O

Figure A.7. Excel Screenshot of Rostered Schedule

The schedules were labeled with the shift code (D=day, E=evening, N=night, O=off) using the following formulas.



For employees 1 thru 8 assigned to cells B83:O90:

**B83 = IF(B2+C2>0,"D",IF(D2+E2>0,"E",IF(F2+G2>0,"N","O")))**

For employees 9 thru 20 assigned to cells B91:O102:

**B91 = IF(B12>0,"D",IF(C12>0,"E",IF(D12>0,"N","O")))**

## Bibliography

1. Bard J., Purnomo H. "Hospital-wide Reactive Scheduling of Nurses with Preference Considerations," *IIE Transactions*, 37: 589-608 (2005)
2. Cheang B., Li H., Lim A., and Rodrigues B. "Nurse Rostering Problems—A Bibliographic Survey," *European Journal of Operational Research*, 151: 447-460 (2003).
3. Davenport A. "Managing Uncertainty in Scheduling: A Survey," <http://www.sintef.no/static/am/opti/kollokvier/1999/presentations/uncertainty-surveys.ps> (18 Dec 2006)
4. Eriksen W., Bruusgaard D., and Knardahl S. "Work Factors as Predictors of Sickness Absence: A Three Month Prospective Study of Nurses' Aides," *Occupational Environmental Medicine*, 60: 271-278 (2003).
5. Ernst A., Jiang H., Krishnamoorthy M., and Sier D. "Staff Scheduling and Rostering: A Review of Applications, Methods, and Models," *European Journal of Operational Research*, 153: 3 - 27 (2004).
6. Knighton S. *An Optimal Network-Based Approach to Scheduling and Re-Rostering Continuous Heterogeneous Workforces*. PhD Dissertation. Arizona State University, Tempe AZ, August 2005.
7. McClave J., Benson P., and Sinchich T. *Statistics for Business and Economics* (Ninth Edition). Upper Saddle River, New Jersey: Pearson Prentice Hall, 2005.
8. Moz M., and Pato M. "A Genetic Algorithm Approach to a Nurse Rerostering Problem," *Computers & Operations Research*, 34: 667-691 (2007).
9. Oliver S. Registered Nurse for Cummings Healthcare Facility, Bangor ME. Telephone Interview. 11 Jan 2007.
10. Ritchie K., Macdonald E., Gilmour W., and Murray K. "Analysis of Sickness Absence Among Employees of Four NHS Trusts," *Occupational Environmental Medicine*, 56: 702-708 (1999).
11. Schaefer A., Johnson E., Kleywegt A., and Nemhauser G. "Airline Crew Scheduling Under Uncertainty," *Transportation Science*, 39.3: 340-348 (August, 2005)

12. Shebalov S., and Klabjan D. "Robust Airline Crew Pairing: Move-up Crews." *Transportation Science*, 40.3: 300-312 (August, 2006)
13. Siferd S., and Benton W. "Workforce Staff and Scheduling: Hospital Nursing Specific Models," *European Journal of Operational Research*, 60: 223-246 (1992).
14. Whitehead D. "Workplace Health Promotion: The Role and Responsibility of Health Care Managers," *Journal of Nursing Management*, 14: 59-68 (2006).

## **Vita**

Captain Paul K. Tower graduated from Franklin High School in Franklin, New Hampshire. He entered undergraduate studies at the University of New Hampshire in Durham, New Hampshire where he graduated with a Bachelor of Science degree in Mechanical Engineering in December 1998. He was commissioned through the Detachment 475 AFROTC at the University of New Hampshire where he was nominated for a Regular Commission.

His first assignment was at Pope AFB, North Carolina, as an aircraft maintenance officer for the 23d Maintenance Squadron and the 74th Fighter Squadron. In February 2002, he was reassigned to the 56th Fighter Wing at Luke AFB, Arizona, where he served as the Officer-In-Charge of the 61st Aircraft Maintenance Unit. He also served as a flight commander in the 56th Equipment Maintenance Squadron and the 56th Component Maintenance Squadron. In August 2005, he entered the Graduate School of Engineering and Management, Air Force Institute of Technology. Upon graduation, he will be assigned to the Air Combat Command staff at Langley AFB, Virginia.

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